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NONDISSIPATIVE INELASTIC STRAIN FOR A SOLID ELEMENT

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UDC 539.3

Elastic strains for a solid element are the part of strain for the element which disappears after it is unloaded (removal of external effects). Inelastic (residual) strains are the part of strain for the element which remains in it after it is unloaded. Apart from inelastic strains for an element with which mechanical energy is converted into heat, nondissipative inelastic strains are possible, i.e., those with which mechanical energy is not converted into heat.

One of the simplest and graphic examples of a deformation process with nondissipative inelastic strains may be deformation of a system of two elastic springs and a rod (Fig. 1 from [1]). During deformation of this system mechanical energy is not converted into heat, but due to overall clamps A the unloading process proceeds in a different way from the loading process, as a result of which the relationship between force p applied to the rod and displacement Δ of the rod will have the form indicated in Fig. 2, where Δ^* is nondissipative inelastic strain of the system.

In this work equations are formulated determining nondissipative inelastic strains for a solid element.

<u>1. Division of Strain into Elastic and Inelastic</u>. As a strain tensor we take [2]

$$\varepsilon_{ij}e^ie^j = \widehat{\varepsilon}_{\alpha\beta}\partial^\alpha\partial^\beta,\quad \widehat{\varepsilon}_{\alpha\beta} = \frac{1}{2}(\partial_\alpha\cdot\partial_\beta - e_\alpha\cdot e_\beta) \tag{1.1}$$

(∂^α and e^i are basis vectors of Lagrangian and Cartesian coordinate systems).

The state of a material element from which strain is reckoned is called the initial state. We assume that stresses in the initial state equal zero, density and temperature equal prescribed values ρ_0 and T_0 differing from zero, and in any stage of the deformation process for the material element it is possible to "unload it completely" to a state with stress and temperature T_0 equal to zero (by cutting an element from a material, heating and cooling it to the temperature of the initial condition, and giving it the possibility of deforming freely).

In gaseous media equality of stresses to zero is only possible with density equal to zero. In this case as an initial condition we take that in which the average stress and temperature equal a prescribed value differing from zero, and by "complete unloading" we

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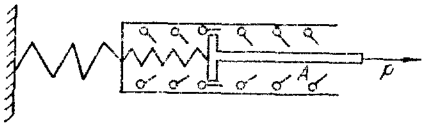

Fig. 1

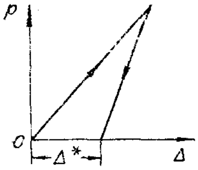

Fig. 2

understand transfer to a condition with average stress and temperature equal to their values in the initial condition.

We designate in terms of $Э_\alpha^*, Э_*^\alpha, \hat{\varepsilon}_{\alpha\beta}^*$ Lagrangian basis vectors and strain tensor components in the "complete unloading" condition. Following [2], we call $\gamma_{ij}\mathbf{e}^i\mathbf{e}^j = \hat{\gamma}_{\alpha\beta}Э^\alpha Э^\beta$, $\hat{\gamma}_{\alpha\beta} = \frac{1}{2}(Э_\alpha \cdot Э_\beta - Э_\alpha^* \cdot Э_\beta^*)$, the elastic strain tensor. The difference in tensors $\varepsilon_{ij}^*\mathbf{e}^i\mathbf{e}^j = \hat{\varepsilon}_{\alpha\beta}^* Э^\alpha Э^\beta$, $\hat{\varepsilon}_{\alpha\beta}^* = (\hat{\varepsilon}_{\alpha\beta} - \hat{\gamma}_{\alpha\beta})/2$, we call the inelastic strain tensor. Thus, in the general case of deformation of a material element components ε_{ij} of the strain tensor are in the form of the sum of components γ_{ij}, ε_{ij}^* for elastic and inelastic strain tensors

$$\varepsilon_{ij} = \gamma_{ij} + \varepsilon_{ij}^*. \tag{1.2}$$

By differentiating with respect to time equations for the link between strain tensor components $\hat{\varepsilon}_{\alpha\beta}$ and ε_{ij} and using the equality

$$\frac{d\hat{\varepsilon}_{\alpha\beta}}{dt} Э^\alpha Э^\beta = e_{ij}\mathbf{e}^i\mathbf{e}^j, \quad e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x^j} + \frac{\partial u_j}{\partial x^i}\right),$$

$$\varepsilon_{ij}\left(\mathbf{e}^i \cdot \frac{dЭ_\alpha}{dt}\right) = \varepsilon_{sj}\frac{\partial u^s}{\partial x^i}(\mathbf{e}^i \cdot Э_\alpha),$$

we obtain

$$\frac{d\varepsilon_{ij}}{dt} = e_{ij} - \varepsilon_i^k \frac{\partial u_k}{\partial x^j} - \varepsilon_j^k \frac{\partial u_k}{\partial x^i} \tag{1.3}$$

where t is time; x^i and u^i are Cartesian coordinates and vector components for particle velocity.

From (1.2) and (1.3) it follows that

$$\frac{d\gamma_{ij}}{dt} = e_{ij} - \gamma_i^k \frac{\partial u_k}{\partial x^j} - \gamma_j^k \frac{\partial u_k}{\partial x^i} - \varphi_{ij}; \tag{1.4}$$

$$\varphi_{ij} = \frac{d\varepsilon_{ij}^*}{dt} + \varepsilon_{ik}^* \frac{\partial u^k}{\partial x^j} + \varepsilon_{jk}^* \frac{\partial u^k}{\partial x^i}. \tag{1.5}$$

Tensor $\varphi_{ij}\mathbf{e}^i\mathbf{e}^j = \frac{d\hat{\varepsilon}_{\alpha\beta}^*}{dt} Э^\alpha Э^\beta$ is called the inelastic strain rate tensor. We designate in terms of γ, e, and φ the first invariants of the elastic strain tensor, strain rates, and inelastic strain rates $\gamma = \sigma^{ij}\gamma_{ij}$, $e = \delta^{ij}e_{ij}$, $\varphi = \delta^{ij}\varphi_{ij}$. From (1.4) it follows that

$$\frac{d\gamma}{dt} = \left(1 - \frac{2}{3}\gamma\right)e - 2\gamma_{ij}' e'^{ij} - \varphi; \tag{1.6}$$

$$\frac{d\gamma_{ij}'}{dt} = \left(1 - \frac{2}{3}\gamma\right)e_{ij}' - \gamma_{jk}' \frac{\partial u^k}{\partial x^i} - \gamma_{ik}' \frac{\partial u^k}{\partial x^j} + \frac{2}{3}\delta_{ij}\gamma_{\nu\mu}' e'^{\nu\mu} - \varphi_{ij}'. \tag{1.7}$$

Here γ_{ij}', e_{ij}', φ_{ij}' are elastic strain deviator components, strain rates, and inelastic strain rates $\gamma_{ij}' = \gamma_{ij} - \frac{1}{3}\delta_{ij}\gamma$, $e_{ij}' = e_{ij} - \frac{1}{3}\delta_{ij}e$, $\varphi_{ij}' = \varphi_{ij} - \frac{1}{3}\delta_{ij}\varphi$.

2. Division of Strains into Dissipative and Nondissipative. We limit ourselves to the case when internal energy U is only a function of elastic strains γ_{ij} and entropy S:

$$U = U(\gamma_{ij}, S). \tag{2.1}$$

Then the energy conservation rule is written in the form

$$\frac{\partial U}{\partial\gamma_{ij}}\left(e_{ij}-\gamma_{ik}\frac{\partial u^k}{\partial x^j}-\gamma_{jk}\frac{\partial u^k}{\partial x^i}-\varphi_{ij}\right)+T\frac{dS}{dt}=\frac{1}{\rho}\left(\sigma^{ij}e_{ij}-\operatorname{div}\mathbf{q}\right),$$

where ρ is density; $\mathbf{q}$ is heat flow vector; T is temperature ($T = \partial U/\partial S$). From the second rule of thermodynamics it is necessary that

$$D=T\frac{dS}{dt}+\frac{1}{\rho}\operatorname{div}\mathbf{q}\geqslant 0 \tag{2.2}$$

(D is dissipative capacity).

Inelastic strain rates are presented in the form of a sum

$$\varphi_{ij}=\psi_{ij}+\varphi_{ij}^{(1)}. \tag{2.3}$$

Here ψ_{ij} are inelastic strain rates on which D depends; $\varphi_{ij}^{(1)}$ are inelastic strain rates on which D does not depend. Division of inelastic strain rates into dissipative and nondissipative corresponds to the equations

$$\varepsilon_{ij}^{*}=\varepsilon_{ij}^{**}+\varepsilon_{ij}^{*(1)},\quad \frac{d\varepsilon_{ij}^{**}}{dt}=\psi_{ij}-\varepsilon_{ik}^{**}\frac{\partial u^k}{\partial x^j}-\varepsilon_{jk}^{**}\frac{\partial u^k}{\partial x^i},$$

$$\frac{d\varepsilon_{ij}^{*(1)}}{dt}=\varphi_{ij}^{(1)}-\varepsilon_{ik}^{*(1)}\frac{\partial u^k}{\partial x^j}-\varepsilon_{jk}^{*(1)}\frac{\partial u^k}{\partial x^i}$$

determined according to (1.5) for the division of inelastic strains ε_{ij}^{*} into dissipative strains ε_{ij}^{*} and nondissipative strains $\varepsilon_{ij}^{*(1)}$.

It is assumed that elastic strains for an element of material are nondissipative. Then thermodynamic rules permit any values of ψ_{ij} and $\varphi_{ij}^{(1)}$ with which the following conditions are fulfilled

$$D=\frac{\partial U}{\partial\gamma_{ij}}\psi_{ij}\geqslant 0; \tag{2.4}$$

$$\frac{\partial U}{\partial\gamma_{ij}}\left(e_{ij}-\gamma_{ik}\frac{\partial u^k}{\partial x^j}-\gamma_{jk}\frac{\partial u^k}{\partial x^i}-\varphi_{ij}^{(1)}\right)-\frac{1}{\rho}\sigma^{ij}e_{ij}=0. \tag{2.5}$$

<u>3. Equations for Deformation of a Material Element with Nondissipative Inelastic Strains</u>. We present nondissipative inelastic strain rates as the sum

$$\varphi_{ij}^{(1)}=C_{ij}^{ks}\frac{\partial u^k}{\partial x^s}+\varphi_{ij}^{(2)}, \tag{3.1}$$

where C_{ij}^{ks} are arbitrary functions of elastic strains and other deformation parameters satisfying the condition $C_{ij}^{ks} = C_{ji}^{ks}$ necessary for symmetry of the strain tensor; $\varphi_{ij}^{(2)}$ are arbitrary functions satisfying the condition $\varphi_{ij}^{(2)}=\varphi_{ji}^{(2)}$.

By substituting (3.1) in (2.5) we find that (2.5) has the form

$$\left[\frac{1}{\rho}\sigma^{ij}-\left(\frac{\partial U}{\partial\gamma_{ij}}-2\frac{\partial U}{\partial\gamma_{ik}}\gamma_k^j-\frac{\partial U}{\partial\gamma_{ks}}C_{ks}^{ij}\right)\right]\frac{\partial u_i}{\partial x^j}+\frac{\partial U}{\partial\gamma_{ij}}\varphi_{ij}^{(2)}=0. \tag{3.2}$$

Equality (3.2) will be fulfilled with arbitrary $\partial u_i/\partial x^j$ if the dependence of stresses on elastic strains, entropy, and function C_{ks}^{ij} is determined by the equalities

$$\sigma^{ij}=\rho\left(\frac{\partial U}{\partial\gamma_{ij}}-2\frac{\partial U}{\partial\gamma_{ik}}\gamma_k^j-\frac{\partial U}{\partial\gamma_{ks}}C_{ks}^{ij}\right) \tag{3.3}$$

and it is demanded that inelastic strain rates $\varphi_{ij}^{(2)}$ satisfy the condition

$$\frac{\partial U}{\partial\gamma_{ij}}\varphi_{ij}^{(2)}=0. \tag{3.4}$$

It is evident that function (2.1) may be considered as a function of γ, γ_{ij}', S and consequently equality (3.4) is written as

$$\varphi^{(2)}=-\frac{\partial U}{\partial\gamma_{ij}'}\varphi_{ij}'^{(2)}\Big/\frac{\partial U}{\partial\gamma},\quad \varphi^{(2)}=\delta^{ij}\varphi_{ij}^{(2)},\quad \varphi_{ij}'^{(2)}=\varphi_{ij}^{(2)}-\frac{1}{3}\delta_{ij}\varphi^{(2)}. \tag{3.5}$$

Thus, (3.4) may be interpreted as condition (3.5) governing the dependence of part $\varphi^{(2)}$ of the volumetric strain rate on parts $\varphi_{ij}'^{(2)}$ of shear strain rates. In this way it is possible for arbitrary dependence of $\varphi_{ij}'^{(2)}$ on elastic strain and other deformation parameters satisfying the condition

$$\varphi_{ij}'^{(2)} = \varphi_{ji}'^{(2)},\ \delta^{ij}\varphi_{ij}'^{(2)} = 0. \tag{3.6}$$

Equations (3.3) contain density. Therefore, it is necessary to add to them a continuity equation

$$d\rho/dt = -\rho e, \tag{3.7}$$

governing the dependence of density on strain rate.

If the dependence of dissipative inelastic strain rates on elastic strains and other deformation parameters is prescribed, Eqs. (1.4), (2.2)-(2.4), (3.1), (3.3), (3.5)-(3.7) form a model for the solid in which apart from elastic and dissipative inelastic strains there may be nondissipative inelastic strains.

4. Nondissipative Inelastic Strains Connected with Simplication of Elastic Deformation Equations. We consider deformation equations with internal energy

$$U = f + T_0\theta + \frac{2\mu}{\rho_0}\Gamma + cT,\ f = f(\gamma),\ \theta = \theta(\gamma),\ \Gamma = \frac{1}{2}\gamma_{ij}'\gamma'^{ij}. \tag{4.1}$$

Here T_0, μ, ρ_0, and c are constants; T is elastic strain function and entropy S is determined by the equation

$$c\frac{dT}{dt} = T\frac{d}{dt}(S - \theta). \tag{4.2}$$

By assuming that in (1.6) and (1.7) $\varphi = \varphi_{ij}' = 0$ and assuming that elastic strains are nondissipative, from the energy conservation rule we find that in the case of (4.1) and (4.2) equations for elastic deformation of a material element are written in the form

$$\sigma = \rho\left(1 - \frac{2}{3}\gamma\right)\frac{\partial U}{\partial\gamma} - \underline{\frac{8\mu\rho}{3\rho_0}\Gamma},\quad \frac{\partial U}{\partial\gamma} = \frac{df}{d\gamma} - (T - T_0)\frac{d\theta}{d\gamma}; \tag{4.3}$$

$$\sigma_{ij}' = \frac{2\mu\rho}{\rho_0}\left(1 - \frac{2}{3}\gamma\right)\left[1 - \frac{\rho_0}{\mu\left(1 - \frac{2}{3}\gamma\right)}\frac{\partial U}{\partial\gamma}\right]\gamma_{ij}' - \frac{4\mu\rho}{\rho_0}\left(\gamma_i'^{\nu}\gamma_{j\nu}' - \frac{2}{3}\delta_{ij}\Gamma\right); \tag{4.4}$$

$$\frac{d\gamma}{dt} = \left(1 - \frac{2}{3}\gamma\right)e - 2\gamma_{ij}'e'^{ij},\quad \frac{d\rho}{dt} = -\rho e; \tag{4.5}$$

$$\frac{d\gamma_{ij}'}{dt} = \left(1 - \frac{2}{3}\gamma\right)e_{ij}' - \gamma_{jk}'\frac{\partial u^k}{\partial x^i} - \gamma_{ik}'\frac{\partial u^k}{\partial x^j} + \frac{2}{3}\delta_{ij}\gamma_{\nu\mu}'e'^{\nu\mu}. \tag{4.6}$$

In many solids components γ_{ij}' of the elastic strain deviator are small compared with unity, for example in metal bodies they are values of the order of the ratio of yield strength to Young's modulus. In these bodies the term underlined in (4.3) is small and it should be discarded, i.e., (4.3) should be substituted by the equations

$$\sigma = \rho\left(1 - \frac{2}{3}\gamma\right)\frac{\partial U}{\partial\gamma},\ \frac{\partial U}{\partial\gamma} = \frac{\partial f}{\partial\gamma} - (T - T_0)\frac{d\theta}{d\gamma}. \tag{4.7}$$

For the same reason (4.4) should be substituted by the equations

$$\sigma_{ij}' = \frac{2\mu\rho}{\rho_0}\left(1 - \frac{2}{3}\gamma\right)\left[1 - \frac{\rho_0}{\mu\left(1 - \frac{2}{3}\gamma\right)}\frac{\partial U}{\partial\gamma}\right]\gamma_{ij}'. \tag{4.8}$$

However, this substitution leads to system (4.5)-(4.8) which will not be thermodynamically correct (it will not satisfy the energy conservation rule). Even when infringement of the energy conservation rule is small, and consequently it does not lead to any marked physical contradictions, the very fact that the energy conservation rule does not emerge from the set of equations may for example markedly complicate formulation and substantiation of algorithms for numerical solution of the problem using this system.

In order that with substitution of (4.3) and (4.4) by Eqs. (4.7) and (4.8) thermodynamic correctness is retained for the set of deformation equations, it is necessary to change

(4.6) also with this substitution. A change of (4.6) means introducing into (1.7) corresponding components φ_{ij}' for the inelastic strain rate deviator. By placing in (1.7)

$$\varphi_{ij}' = -\gamma_{ik}' e_j^{k} - \gamma_{jk}' e_i^{k} + \frac{2}{3}\sigma_{ij}\gamma_{\alpha\beta}' e'^{\alpha\beta}, \tag{4.9}$$

we find that derived components γ_{ij}' of the elastic strain deviator with respect to time in the case of (4.9) will be connected with strain rates and rotation angles by the equations

$$\frac{d\gamma_{ij}'}{dt} = \left(1 - \frac{2}{3}\gamma\right) e_{ij}' - \gamma_j'^{k}\omega_{ki} - \gamma_i'^{k}\omega_{kj},$$
$$\omega_{ki} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x^i} - \frac{\partial u_i}{\partial x^k}\right).$$

It is easy to be certain that system (4.5), (4.7), (4.8), (4.10) is thermodynamically correct. It satisfies the energy conservation rule with dissipative capacity equal to zero, and therefore inelastic strain rates determined by Eqs. (4.9) are nondissipative elastic strain rates. If these rates are small compared with strain rates e_{ij}, the deformation process for (4.5), (4.7), (4.8), (4.10) may be considered elastic.

The term $\rho_0/\mu(1 - 2\gamma/3)\cdot\partial U/\partial\gamma$ in (4.8) may be small in many cases (of the order of the ratio of average stress σ to elastic shear modulus μ). A condition for retaining thermodynamic correctness of the set of deformation equations for a material element with simplification of (4.8) by substituting them by equations

$$\sigma_{ij}' = \frac{2\mu\rho}{\rho_0}\left(1 - \frac{2}{3}\gamma\right)\gamma_{ij}' \tag{4.11}$$

requires substitution of (4.5) by equations

$$\frac{d\gamma}{dt} = \left(1 - \frac{2}{3}\gamma\right)e, \quad \frac{d\rho}{dt} = -\rho e, \tag{4.12}$$

i.e., introduction into (1.6) of rates for nondissipative elastic change in volume $\varphi = -2\gamma_{ij}' e'^{ij}$.

Similarly it is possible to carry out further simplification of system (4.7), (4.10)-(4.12) by introducing the corresponding nondissipative inelastic strains. For example, by placing in (1.6) and (1.7)

$$\begin{aligned}\varphi &= \left(1 - \frac{2}{3}\gamma - \frac{\rho_0}{\rho}\right)e - 2\gamma_{ij}' e'^{ij},\\ \varphi_{ij}' &= \left(1 - \frac{2}{3}\gamma - \frac{\rho_0}{\rho}\right)e_{ij}' - \gamma_j'^{k} e_{ki} - \gamma_i'^{k} e_{kj} + \frac{2}{3}\delta_{ij}\gamma_{\alpha\beta}' e'^{\alpha\beta},\end{aligned} \tag{4.13}$$

we obtain

$$\begin{aligned}&\sigma_{ij}' = 2\mu\gamma_{ij}', \ \sigma = \rho_0\frac{\partial U}{\partial\gamma}, \ \frac{\partial U}{\partial\gamma} = \frac{df}{d\gamma} - (T - T_0)\frac{d\theta}{d\gamma},\\ &\frac{d\gamma_{ij}'}{dt} = \frac{\rho_0}{\rho}e_{ij}' - \gamma_i'^{k}\omega_{kj} - \gamma_j'^{k}\omega_{ki}, \ \frac{d\gamma}{dt} = \frac{\rho_0}{\rho}e, \ \frac{d\rho}{dt} = -\rho e.\end{aligned} \tag{4.14}$$

With a small change in density, conditions for thermodynamic correctness of the equations may be weakened by substituting them by approximate conditions

$$\frac{dU}{dt} = \frac{1}{\rho_0}\left(\sigma^{ij}e_{ij} - \operatorname{div}\mathbf{q}\right), D = T\frac{dS}{dt} + \frac{1}{\rho_0}\operatorname{div}\mathbf{q} \geqslant 0. \tag{4.15}$$

Equations (1.6), (1.7) and conditions (4.15) will be fulfilled if in (4.13) and (4.14) ρ equal to ρ_0 is placed.

5. Nondissipative Inelastic Strains Connected with a Change in Elasticity "Moduli." Simplification of elastic deformation equations is only one of the reasons for appearance of inelastic nondissipative strains in equations for a solid. Another reason may be introduction into deformation equations for a material element of nondissipative inelastic strains with the aim of describing different phenomena, such as for example the difference in resis-

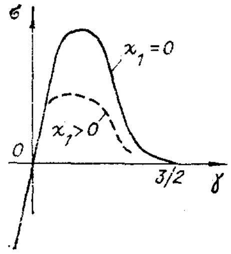

Fig. 3

tance of a material element to deformation and failure in tension and compression, embrittlement (reduction during deformation of resistance to brittle failure), loosening (densification) with shear strains, strain localization during failure, etc.

Possibilities for describing these phenomena by introducing nondissipative inelastic strains are considered on the example of a material in which internal energy is prescribed in the form of equalities (4.1) and (4.2), and apart from elastic strains there may only be nondissipative inelastic strains with rates in the form

$$\begin{gathered}\varphi = \chi_1 e + 2\chi_2 \gamma'_{ij} e'^{ij},\\ \varphi'_{ij} = \chi_3 e'_{ij} + \frac{2}{3}\delta_{ij}\gamma'_{\alpha\beta} e'^{\alpha\beta} - \gamma'^{k}_{j} e_{ki} - \gamma'^{k}_{i} e_{kj}\end{gathered} \tag{5.1}$$

(χ_k, k = 1, 2, 3 are arbitrary functions of elastic strains and other deformation parameters). In the case in question derived elastic strains in time are connected according to (1.6) and (1.7) with strain rates by equalities

$$\begin{gathered}\frac{d\gamma}{dt} = \left(1 - \frac{3}{2}\gamma - \chi_1\right) e - 2(1 + \chi_2)\gamma'_{\alpha\beta} e'^{\alpha\beta},\\ \frac{d\gamma'_{ij}}{dt} = \left(1 - \frac{2}{3}\gamma - \chi_3\right) e'_{ij} - \gamma'^{k}_{i}\omega_{kj} - \gamma'^{k}_{j}\omega_{ki},\end{gathered} \tag{5.2}$$

and Eq. (3.3) are written as

$$\begin{gathered}\sigma = \rho\left(1 - \frac{2}{3}\gamma - \chi_1\right)\frac{\partial U}{\partial \gamma},\ \sigma'_{ij} = 2\tilde{\mu}\gamma'_{ij},\\ \frac{\partial U}{\partial \gamma} = \frac{df}{d\gamma} - (T - T_0)\frac{d\theta}{d\gamma},\ f = f(\gamma),\ \theta = \theta(\gamma),\\ \tilde{\mu} = \mu\frac{\rho}{\rho_0}\left[1 - \frac{2}{3}\gamma - \chi_3 - (1 + \chi_2)\frac{\rho}{\mu}\frac{\partial U}{\partial \gamma}\right].\end{gathered} \tag{5.3}$$

We consider overall expansion of a material element under conditions that $\chi_1 = 0$, $T = T_0$. In this way $e'_{ij} = \omega_{ij} = 0$, $e = -\frac{1}{\rho}\frac{d\rho}{dt}$, $\frac{d\gamma}{dt} = \left(1 - \frac{2}{3}\gamma\right)e$ and consequently

$$\rho = \rho_0\left(1 - \frac{2}{3}\gamma\right)^{3/2}. \tag{5.4}$$

In formulating deformation equations for a solid with a marked change in density it is normally assumed (see e.g., [3]) that internal energy for the element with $\rho \to 0$ tends asymptotically toward some constant value similar to the reaction energy for atoms with a change in distance between them to infinity. Assuming that function $f(\gamma)$ in (5.3) satisfies this condition, from (5.3) and (5.4) we find that

$$\sigma \to 0 \text{ with } \gamma \to 3/2. \tag{5.5}$$

With small values of γ the dependence of σ on γ is close to linear

$$\sigma = A\gamma \tag{5.6}$$

(A is a positive constant). It follows from (5.5) and (5.6) that in the case of overall expansion with $\chi_1 = 0$, $T = T_0$ the curve for the dependence of σ on γ has the form shown in Fig. 3 by the solid curve. It is evident that it will be the same in the general case with a deformation of a material element with $\chi_1 = 0$, $T = T_0$. Its characteristic feature is the limitedness of average stress. Presence of function χ_1 in (5.3) creates the possibility of changing by means of function χ_1 the dependence of σ on γ, as for example shown by a broken line in Fig. 3. In particular, this makes it possible to describe by means of function χ_1 a change for one reason or another of material element resistance to failure due to the limitedness of average stress.

The presence of function χ_2 in (5.3) makes it possible by using it to describe the effect of shear strains on the change in volume and average stress. Let deformation proceed with $\sigma = \gamma = 0$. In this case, according to (5.2) shear strains lead to a change in volume by the equation $e = \frac{2(1+\chi_2)}{1-\chi_1}\gamma'_{\alpha\beta}e'^{\alpha\beta}$. Let deformation occur without a change in volume. Then shear strains lead to a change in γ by the equation $\frac{d\gamma}{dt} = -2(1+\chi_2)\gamma'_{\alpha\beta}e'^{\alpha\beta}$ and consequently to a corresponding change in average stress. Presence of function χ_3 in (5.3) makes it possible by using it to describe the change for one reason or another of elastic shear modulus $\tilde{\mu}$.

The examples provided demonstrate that there are very extensive possibilities for describing different phenomena by introducing nondissipative inelastic strains into deformation equations for a material element.

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DIRECT METHOD OF DETERMINING THE DYNAMICAL RESPONSE IN INTERACTIONS OF CONSTRUCTION ELEMENTS WITH CONCENTRATED MASSES AND RIGIDITIES

A. V. Agafonov UDC 534.1

In many cases of calculation of construction elements (rods, plates, etc.) subjected to concentrated interactions it is sufficient to know only the strain-stress state of the considered element directly at the points of load and the element on the whole (or the whole construction) is of interest only in the sense of its integral response to the interaction.

If the concentrated interaction is given then finding this integral response is usually not difficult. In the same cases when the interaction depends on the motion of the construction element itself, determining the integral response necessitates coupling the variation of the load to the motion of the construction.

To solve such problems one uses the basic method of dynamic susceptibilities [1, 2]. According to this method the solution is constructed in two stages [1]: first, one finds separately the dynamic susceptibilities of the element and the mass (rigidity) acting on it under action of a suddenly applied concentrated force; next, one seeks the response to the interaction of the element with the mass (rigidity) from the integrodifferential equation expressing the condition of equality of the displacements of the mass and the element at the point of interaction.

At the same time there exists a possibility of developing a method which permits one to determine parameters of interest at the point of interaction bypassing the preliminary determination of the dynamic susceptibilities and shortening the process of solving the problem. Such a method can be proposed on the basis of integral transforms and the formalism of δ-functions.

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